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# Mean estimation based on the factor-type estimator under an adaptive cluster sampling design

**Narendra Singh Thakur[1], Shubhangi Chaurasia[2], Unnati Bhayare[3]**

## Abstract

If a sample is designated by a standard sampling strategy and if the character of the study satisfies a predetermined statement for an independent unit in the sample, then the items in the locality remain automatically in the sample. This type of method of selection of sampling units is called adaptive cluster sampling. This manuscript emphasizes the use of the factor-type estimator designed for population mean of the variable under study using the data of highly correlated auxiliary (supplementary) variable under adaptive cluster sampling. The bias, mean squared error and optimum mean squared errors up to the first order is obtained and a simulation study is performed for comparison purpose.

**Key words:** adaptive cluster sampling (ACS), ratio estimator, factor-type estimator, auxiliary variable, bias, mean squared error (MSE).

**Mathematical Subject Code**: 62D05.

## 1. Introduction

Thompson (1990) introduced an innovative sampling scheme called adaptive sampling, which directly incorporates the knowledge of the study variable into the selection process. This approach is distinct from traditional sampling strategies that rely solely on predetermined sampling plans. The adaptive sampling scheme was proposed to address situations where the study variable exhibits certain patterns or characteristics that can inform the sampling process. For instance, in surveys involving rare species, researchers may gather information on the number of individuals with specific characteristics. Frequently, zero abundance is encountered, but when substantial abundance is observed, it suggests that additional clusters of abundance might be found in nearby

[1] Department of Statistics, Govt. Model Girls College, Sheopur, MP, India. E-mail: nst_stats@yahoo.co.in. ORCID: https://orcid.org/0000-0001-9731-058X.
[2] Department of Mathematics and Statistics, SMS Govt. Model Science College, Gwalior, MP, India. E-mail: shubhangichaurasia22101989@gmail.com. ORCID: https://orcid.org/0009-0007-2399-7746.
[3] Department of Statistics, Govt. (Model, Autonomous) Holkar Science College, Indore, MP, India. E-mail: unnati.b80@gmail.com. ORCID: https://orcid.org/0009-0001-5561-0303.

locations. This pattern is not limited to rare species but can be observed in various domains such as whales, insects, trees, lichens, and more. The conventional approach in sample surveys involves deciding on a sampling strategy before data collection begins. However, this predetermined approach may not always be effective, especially in certain scenarios. For instance, in epidemiological studies of infectious diseases, encountering a diseased individual suggests a higher-than-expected incidence rate among nearby individuals. In such cases, ground staff may deviate on or after the predesignated selection plan and then combine adjacent or closely allied items to the sample.

Thompson's (1990) adaptive sampling scheme addresses this need for flexibility in sampling. It starts with drawing a preliminary sample of a predetermined size using a standard sampling strategy. The values of the sampled items are then examined, and if an elected item fulfils a specified condition, supplementary items are put in to the sample from the locality of that item. Thus, adaptive process allows for the expansion of the sample based on specific criteria or patterns observed in the study variable. The design of the adaptive sampling scheme is demonstrated in Figure 1(a) and 1(b), which likely provide visual representations of how the sampling process unfolds.

The Figure 1(a) illustrates the preliminary sampling stage of the adaptive sampling scheme. A sample of 12 units is selected using a probability sampling procedure, which could be any conventional sampling design. The key feature of adaptive sampling is that when one or more units in the preliminary sample satisfy a specific criterion associated to the variable under study, accompanying units from locality of those selected units are included in the sample. The neighborhood is typically defined based on spatial proximity, as indicated by the connected units on the left, right, top, and bottom in Figure 1(a).

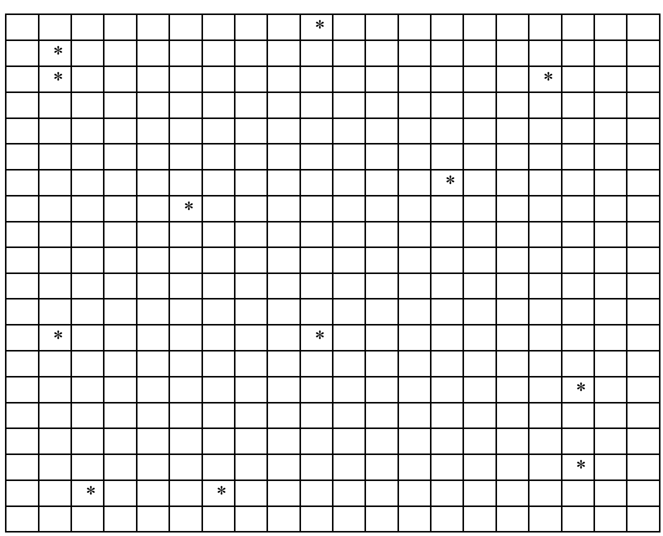

**Figure 1(a):** Preliminary sample of 12 units

After the adaptive procedure is finished, the sample contains 54 units, as revealed in Figure 1(b), where the symbol $\underline{*}$ represents the unit selected in preliminary sample of size 12. It should be noted that the concept of neighborhood is not limited to spatial proximity and is able to be express in numerous aspects subject to the condition and the nature of the study. In summary, in the adaptive sampling scheme, a preliminary subgroup of some units is selected using a probability sampling technique and if the variable of interest for a carefully chosen item satisfies a given criterion, subsequent units from the locality of that item are considered in the sample. This adaptive approach allows for the expansion of the sample based on specific conditions or patterns observed in the variable of interest.

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**Figure 1(b):** Adaptive cluster sample of 54 units

In the adaptive selection system, the criterion for picking additional neighboring items can be defined in various ways, depending on the nature of the study. One approach is to frame the criterion as an interval $L$ that covers a specific range of values related to the variable of curiosity. If a unit is considered in the sample, it should meet the criteria fixed by the interval $L$. Mathematically, it can be represented as if $i \in L$ then the unit $i \in S$.

The provided definitions lay the foundation for understanding the structure and components of the sampling process in the adaptive cluster sampling scheme. Let's elaborate on each definition:

1) **Neighborhood of a unit:** The neighborhood of a unit $i$ refers to a group of items that contains unit $i$. These neighborhoods are determined on the basis of design and selection process and are independent of the population values.

2) **Cluster:** A cluster is the assembly of all units that are detected as per an outcome of the preliminary choice of a specific item $i$. In ACS, the preliminary selection of a unit (seed unit) leads to the insertion of entire units in the corresponding cluster in the final sample. It is possible for a cluster to consist of the union of several neighborhoods, which means that multiple neighborhoods can be grouped together as part of the same cluster. The concept of clusters is important for understanding the unit selection process and how ACS samples are formed.
3) **Network**: It is a set of items where the inclusion of any item in the preliminary sample from that set ensures the inclusion of all units in that network in the final sample. In other words, if a single unit from a network is selected, the entire network becomes part of the sample. It is worth noting that units not satisfying the condition $L$ are also considered network, but they consist of a single unit only. Networks play a significant role in adaptive sampling, where certain networks may be oversampled to improve estimation efficiency.
4) **Edge unit:** An edge unit is a population item that does not satisfy the network requirements but is in the neighborhood of an item that satisfies the condition $L$. Essentially, edge units are on the boundary of clusters or networks. They play a crucial role in ACS because their selection may influence the inclusion of entire clusters or networks in the final sample.

The estimators taken into consideration as a relationship between neighborhoods, clusters, networks, and edge units to obtain a reliable and efficient population estimate and to make a valid statistical inference under adaptive cluster sampling. For further study some useful and valuable contributions for readers are advise as Chao (2004), Chutiman and Kumphon (2008), Dryver and Thompson (1998), Pochai (2008) etc. This manuscript is concerned with to develop the factor-type estimator in adaptive cluster sampling and discuss its properties. Furthermore, how we were motivating for writing this manuscript is explained in Section 3 of this paper.

### 1.1. Notations

Let $y$ be the variable under study based on a population $U$ and let it consists of a set of $N$ units indexed by their labels $U = \{1, 2, \dots N\}$. The population mean of $y$ is $\bar{Y} = \mu_y = \frac{1}{N}\sum_{i=1}^{N} y_i$. Let $\bar{Y}_{ac} = \bar{w}_y = \frac{1}{n}\sum_{i=1}^{n} w_{yi}$ be the estimate of the mean in adaptive cluster sampling. Let us consider:

$n$ = Size of preliminary sample,

$A_i$ = Network which consists of the item $i$,

$m_i$ = The amount of the items in the network to which $i^{th}$ item belongs.

Let $w_{yi}$ and $w_{xi}$ represent the mean of $y$ and the mean of $x$ in the network which consist of unit $i$, viz., $w_{yi} = \frac{1}{m_i}\sum_{j\in A_i} y_j$ and $w_{xi} = \frac{1}{m_i}\sum_{j\in A_i} x_j$ respectively. According to

Dryver and Chao (2007) adaptive cluster sampling is considered as SRSWOR when the means of networks are considered under study. Let us use the notations $\bar{w}_y$ and $\bar{w}_x$ to denote the sample means of the study and supplementary variables in the transformed sample respectively. We calculate $\bar{w}_y$ and $\bar{w}_x$ as

$$\bar{w}_y = \frac{1}{n}\sum_{i=1}^{n} w_{yi} \text{ and } \bar{w}_x = \frac{1}{n}\sum_{i=1}^{n} w_{xi}$$

For simplicity we write,

$$s_{wy}^2 = \frac{1}{n-1}\sum_{i=1}^{n}(w_{yi} - \bar{w}_y)^2,\ s_{wx}^2 = \frac{1}{n-1}\sum_{i=1}^{n}(w_{xi} - \bar{w}_x)^2$$

are unbiased estimators of

$$S_{wy}^2 = \frac{1}{N-1}\sum_{i=1}^{N}(w_{yi} - \bar{Y})^2,\ S_{wx}^2 = \frac{1}{N-1}\sum_{i=1}^{N}(w_{xi} - \bar{X})^2 \quad \text{respectively,}$$

and $\mu_{rs} = \frac{1}{N}\sum_{i=1}^{N}\left(w_{yi} - \bar{Y}\right)^r (w_{xi} - \bar{X})^s$ ; where $r, s$ are positive integers.

Also, $C_{wy}^2 = \frac{S_{wy}^2}{\bar{Y}^2}$ and $C_{wx}^2 = \frac{S_{wx}^2}{\bar{X}^2}$ are the coefficient of variations of $w_y$ and $w_x$ respectively,

and $\rho_{wyx} = \frac{\mu_{11}}{S_{wy}S_{wx}}$ is the coefficient of correlation between $w_y$ and $w_x$.

Using the concept of large sample approximations, let $\varepsilon = \frac{\bar{w}_y}{\bar{Y}} - 1$ and $\eta = \frac{\bar{w}_x}{\bar{X}} - 1$, for specified $\bar{w}_y$, $\bar{w}_x$ respectively, then

$$E(\varepsilon) = E(\eta) = 0,\ E(\varepsilon^2) = \frac{C_{wY}^2}{n},\ E(\eta^2) = \frac{C_{wX}^2}{n},\ E(\varepsilon\eta) = \rho_{wyx}C_{wY}C_{wX}$$

and $E(\varepsilon^i\eta^j) = 0$ if $i + j > 2;\ i, j = 0, 1, 2, \ldots$ .

The expectations as derived above under the concept of large sample approximations will be used for further mathematical treatments.

**Remark:** Assume, $\alpha = \frac{1}{1+C_{wx}^*}$; $\omega = \frac{\beta_2(w_x)}{\beta_2(w_x) + C_{wx}^*}$ ; $\delta = \frac{1}{1+\beta_2^*(w_x)}$ ;

$$\theta_1 = \frac{fB}{A+fB+C};\ \theta_2 = \frac{C}{A+fB+C};\ P = (\theta_1 - \theta_2);\ V = \rho_{wyx}\frac{C_{wy}}{C_{wx}}.$$

The above symbols will be used for further algebraic treatments.

## 2. Existing estimators and their characteristics in adaptive cluster sampling

Some existing estimators under adaptive cluster sampling are considered in this section. Note that, the constants $C_{wx}^* = \frac{C_{wx}}{\bar{X}}$ , $\beta_2^*(w_x) = \frac{\beta_2(w_x)}{\bar{X}}$, $u_{wx} = \frac{\bar{w}_x}{\bar{x}}$ and $\bar{X} = \mu_x = \frac{1}{N}\sum_{i=1}^{N} x_i$ , $C_{wx}$, $\beta_2(w_x)$ of the auxiliary variable are known in advance by past experience.

The unbiased Hansen and Hurwitz estimator for the population mean of main variable is given by [see, Thompson (1990), Thompson and Seber (1996)]

$$\bar{Y}_{ac} = \frac{1}{N}\sum_{i=1}^{N}(w_y)_i = \bar{w}_y \tag{2.1}$$

where, $(w_y)_i$ is mean of the main variable in the network that contains item $i$ of the preliminary sample.

The variance of $\bar{Y}_{ac}$ is

$$V(\bar{Y}_{ac}) = \frac{(N-n)}{Nn(N-1)} \sum_{i=1}^{N} [(w_y)_i - \mu_y]^2 \quad = \frac{N-n}{Nn} S_{wy}^2 \tag{2.2}$$

The ratio type estimator under adaptive cluster sampling strategy projected by Dryver and Chao (2007) as

$$\bar{Y}_{Rac} = \bar{w}_y \frac{\bar{X}}{\bar{w}_x} = \hat{R}_{ac} \bar{X} \tag{2.3}$$

where, $\hat{R}_{ac} = \frac{\bar{w}_y}{\bar{w}_x}$. The estimator $\bar{Y}_{R_{ac}}$ in relationships of $\varepsilon$ and $\eta$ can approximate up to the first order and is expressed as

$$\bar{Y}_{R_{ac}} = \bar{Y}[1 + \varepsilon - \eta - \varepsilon\eta + \eta^2]$$

This estimator is biased and its bias up to the first order is

$$B(\bar{Y}_{R_{ac}}) = \frac{\bar{Y}}{n} \left[C_{wx}^2 - \rho_{wyx} C_{wy} C_{wx}\right] \tag{2.4}$$

The expression of MSE is

$$M(\bar{Y}_{R_{ac}}) = \frac{\bar{Y}^2}{n} \left[C_{wy}^2 + C_{wx}^2 - 2\rho_{wyx} C_{wy} C_{wx}\right] \tag{2.5}$$

Chutiman (2013) suggested some estimators in adaptive cluster sampling described as below

**(A)**

$$\bar{Y}_{R_{ac2}} = \bar{w}_y \left(\frac{1 + C_{wx}^*}{u_{wx} + C_{wx}^*}\right) \tag{2.6}$$

and the estimator $\bar{Y}_{R_{ac1}}$ in relationships of $\varepsilon$ and $\eta$ approximate up to the first order, can be expressed as

$$\bar{Y}_{Rac1} = \bar{Y}[1 + \varepsilon - \alpha\eta - \alpha\varepsilon\eta + \alpha^2\eta^2]$$

This estimator is biased and its bias up to the first order is

$$B\left(\bar{Y}_{R_{ac2}}\right) = \frac{\bar{Y}}{n} \left[\alpha^2 C_{wx}^2 - \alpha\rho_{wyx} C_{wy} C_{wx}\right] \tag{2.7}$$

The equation of MSE is

$$M\left(\bar{Y}_{R_{ac1}}\right) = \frac{\bar{Y}^2}{n} \left[C_{wy}^2 + \alpha^2 C_{wx}^2 - 2\alpha\rho_{wyx} C_{wy} C_{wx}\right] \tag{2.8}$$

**(B)**

$$\bar{Y}_{R_{ac2}} = \bar{w}_y \left[\frac{\beta_2(w_x) + C_{wx}^*}{\beta_2(w_x) u_{wx} + C_{wx}^*}\right] \tag{2.9}$$

and the estimator $\bar{Y}_{R_{ac2}}$ in relationships of $\varepsilon$ and $\eta$ approximate up to the first order, can be expressed as

$$\bar{Y}_{R_{ac2}} = \bar{Y} \left[1 + \varepsilon - \omega\eta - \omega\varepsilon\eta + \omega^2\eta^2\right]$$

This estimator is biased and its bias up to the first order is

$$B\left(\bar{Y}_{R_{ac2}}\right) = \frac{\bar{Y}}{n} [\omega^2 C_{wx}^2 - \omega\rho_{wyx} C_{wy} C_{wx}] \tag{2.10}$$

The equation of MSE is

$$M\left(\bar{Y}_{R_{ac2}}\right) = \frac{\bar{Y}^2}{n} \left[C_{wy}^2 + \omega^2 C_{wx}^2 - 2\omega\rho_{wyx} C_{wy} C_{wx}\right] \tag{2.11}$$

**(C)** $$\bar{Y}_{R_{ac3}} = \bar{w}_y \left[\frac{1+\beta_2^*(w_x)}{u_{wx}+\beta_2^*(w_x)}\right] \tag{2.12}$$

and the estimator $\bar{Y}_{R_{ac3}}$ in relationships of $\varepsilon$ and $\eta$, approximate up to the first order, can be expressed as

$$\bar{Y}_{R_{ac3}} = \bar{Y}\left[1+\varepsilon-\delta\eta-\delta\varepsilon\eta+\delta^2\eta^2\right]$$

This estimator is biased and its bias up to the first order is

$$B\left(\bar{Y}_{R_{ac3}}\right) = \frac{\bar{Y}}{n}\left[\delta^2 C_{wx}^2 - \delta\rho_{wyx}C_{wy}C_{wx}\right] \tag{2.13}$$

The equation of MSE is

$$M\left(\bar{Y}_{R_{ac3}}\right) = \frac{\bar{Y}^2}{n}\left[C_{wy}^2 + \delta^2 C_{wx}^2 - 2\delta\rho_{wyx}C_{wy}C_{wx}\right] \tag{2.14}$$

The proofs of the above expressions are simple and readers can obtain them similar manner to that described in Section 4 for the proposed factor-type estimator using large sample approximations.

## 3. Proposed Estimator in adaptive cluster sampling

Singh and Shukla (1987) proposed a factor-type (F-T) estimator for estimating population mean and Singh and Shukla (1993) derived an efficient factor-type estimator family for estimating the similar population mean. Shukla (2002) suggested factor-type estimator for estimation in two-phase sampling. Also, Shukla and Thakur (2008), Thakur and Shukla (2022) developed factor-type estimator as a device of imputation used for dealing with missingness of the data.

Deriving motivation from all of these, we advocate the modified factor-type estimator for adaptive cluster sampling is

$$\bar{Y}_{FTc} = \bar{w}_y \frac{(A+C)\bar{X}+fB\bar{w}_x}{(A+fB)\bar{X}+C\bar{w}_x} \tag{3.1}$$

where, $A = (k-1)(k-2)$; $B = (k-1)(k-4)$;

$C = (k-2)(k-3)(k-4)$; $f = \frac{n}{N}$ and $0 < k < \infty$ is a constant.

The estimator $\bar{Y}_{FTc}$ is biased and the expressions of bias, MSE and optimum MSE up to the first order of approximations are obtained ahead in Section 4. Theoretical and numerical comparisons of different estimators as discussed earlier, with $\bar{Y}_{FTc}$ is presented in Section 5 and Section 7 respectively.

For some specified values of $k$, the estimator $\bar{Y}_{FTc}$ provides some well-known estimators like – ratio, product, dual to ratio and unbiased unit mean estimator for population mean, i.e. at $k$ = 1, 2, 3 and 4, the estimator $\bar{Y}_{FTc}$ is as special case in the following Table 3.1.

**Table 3.1:** Adaptive factor-type estimator as special cases

| **Value of *k*** | **Estimators** | **Value of *k*** | **Estimators** |
|---|---|---|---|
| $k = 1$ | $\bar{y}_{FTc} = \bar{w}_y \frac{\bar{X}}{\bar{w}_x}$ | $k = 2$ | $\bar{y}_{FTc} = \bar{w}_y \frac{\bar{w}_x}{\bar{X}}$ |
| $k = 3$ | $\bar{y}_{FTc} = \bar{w}_y \frac{N\bar{X} - n\bar{w}_x}{(N-n)\,\bar{X}}$ | $k = 4$ | $\bar{y}_{FTc} = \bar{w}_y$ |

For $k = 1$ the estimator $\bar{y}_{FTc}$ provides the ratio estimator for mean, for $k = 2$ the proposed estimator is termed in product estimator, for $k = 3$ the estimator $\bar{y}_{FTc}$ is converted as dual to ratio estimator and for $k = 4$ the factor of auxiliary information vanishes and the estimator $\bar{y}_{FTc}$ is the same as unbiased unit mean estimator in adaptive cluster sampling.

## 4. Bias, MSE and optimum MSE of the proposed estimator

Let $B(\hat{\theta})$, $M(\hat{\theta})$ and $M(\hat{\theta})_{min}$ represents the bias, MSE and minimum MSE of the estimator $\theta$. Further, the equations of bias, MSE and minimum MSE of the proposed estimators in terms of population parameters and other constants (as available) up to the first order are represented in the subsequent theorems using the concept of large sample approximations.

**Theorem 4.1:** The estimator $\bar{Y}_{FTc}$ in terms of $\varepsilon$ and $\eta$ up to the first order, could be stated as

$$\bar{Y}_{FTc} = \bar{Y}[1 + \varepsilon + P(\eta + \varepsilon\eta - \eta^2\theta_2)] \tag{4.1}$$

**Proof:** From equation (3.1), we have

$$\bar{Y}_{FTc} = \bar{w}_y \ \frac{(A+C)\bar{X} + fB\bar{w}_x}{(A+fB)\bar{X} + C\bar{w}_x}$$

and by using the concept of large sample approximation as discussed in Section 1.1

$$\bar{Y}_{FTc} = \bar{\text{Y}}\ (1+\varepsilon)\left[\frac{(A+C)\bar{X} + fB\bar{X}(1+\eta)}{(A+fB)\bar{X} + C\bar{X}\ (1+\eta)}\right]$$

$$= \bar{Y}(1+\varepsilon)(1+\theta_1\eta)\left(1+\theta_2{}^{-1}\right)$$

$$= \bar{Y}(1+\varepsilon)(1+\theta_1\eta)(1+\theta_2\eta+\theta_2^2\eta^2-\cdots)$$

$$= \bar{Y}[1 + \varepsilon + P(\eta + \varepsilon\eta - \eta^2\theta_2)]$$

**Theorem 4.2:** Bias of $\bar{Y}_{FTc}$ in the relationships of population parameters is

$$B(\bar{Y}_{FTc}) = -\frac{\bar{Y}}{n} P[\theta_2 C_{wx}^2 + \rho_{wyx} C_{wy} C_{wx}] \tag{4.2}$$

**Proof:** $B(\bar{Y}_{FTc}) = E(\bar{Y}_{FTc} - \bar{Y})$

$$= -\frac{\bar{Y}}{n} P\left[\theta_2 C_{wx}^2 - \rho_{wyx} C_{wy} C_{wx}\right]$$

**Theorem 4.3:** MSE of $\bar{Y}_{FTc}$ in the relationships of population parameters is

$$M(\bar{Y}_{FTc}) = \frac{\bar{Y}^2}{n}[C_{wy}^2 + P^2 C_{wx}^2 + 2P\rho_{wyx} C_{wy} C_{wx}] \tag{4.3}$$

**Proof:** $M(\bar{Y}_{FTc}) = E(\bar{Y}_{FTc} - \bar{Y})^2$

$$= \frac{\bar{Y}^2}{n}\left[C_{wy}^2 + P^2 C_{wx}^2 + 2P\rho_{wyx}C_{wy}C_{wx}\right]$$

**Theorem 4.4:** The minimum MSE of $(\bar{Y}_{FTc})$, when $P = -V$ is

$$M(\bar{Y}_{FTc})_{min} = \frac{S_{wy}^2}{n}(1 - \rho_{wyx}^2) \tag{4.4}$$

**Proof:** Minimum MSE occurs when

$$\frac{d}{dP}M(\bar{Y}_{FTc}) = 0$$

or

$$2PC_{wx}^2 + 2\rho_{wyx}C_{wy}C_{wx} = 0$$

the optimal condition is

$$P = -\rho_{wYX}\frac{C_{wy}}{C_{wx}} = -V \qquad \text{(let)} \tag{4.5}$$

Hence, $\mathrm{M}(\bar{Y}_{FTc})_{min} = \left(1 - \rho_{wyx}^2\right)\frac{S_{wy}^2}{n}$

By simplifying the optimality condition $P = -\rho_{wYX}\frac{C_{wy}}{C_{wx}} = -V$, we will get a cubic equation in terms of $k$ and the roots of this equation will provide us best choice of parameter $k$ for minimum mean squared error with lowest bias.

### 4.1. Bias control estimator $\bar{Y}_{FTc}$

The condition of optimality provides from equation (4.5)

$$AV + (V+1)fB + (V-1)C = 0 \tag{4.6}$$

The equation (4.6) is an equation of degree 3 in terms of $k$.

Obviously, at most three values of $k$ ($k_1$, $k_2$, $k_3$) are possible for which MSE is optimum.

The choice criteria for best estimation is:

**1)** Compute

$$\left|B(\bar{Y}_{FTc})_{k_j}\right| \text{ for } j = 1, 2, 3$$

**2)** From computed values, choose $k_j$ as

$$\left|B(\bar{Y}_{FTc})_{k_j}\right| = min\left[\left|B(\bar{Y}_{FTc})_{k_j}\right|\right]; j = 1, 2, 3.$$

So, it is clear that the estimator $\bar{Y}_{FTc}$ is bias control for the optimum MSE.

## 5. Comparisons

This section compares the proposed estimator $\bar{y}_{FTc}$ with existing estimators as discussed in Section 2 of this manuscript. Let $\hat{\theta}_1$, $\hat{\theta}_2$, $\hat{\theta}_3$,…$\hat{\theta}_k$ be $k$ estimators of the population parameter $\theta$ and there exist an estimator $\hat{\theta}$ of the population parameter $\theta$ such that

$$\forall\, i, \quad \mathrm{var}\,(\hat{\theta}) < \mathrm{var}\,(\hat{\theta}_i),\ i = 1, 2, 3, \ldots, k$$

i.e., variance of $\hat{\theta}$ is minimum among all existing estimators $\hat{\theta}_i$, $i$ =1, 2, 3,…, $k$, then $\hat{\theta}$ is best estimator of the population parameter $\theta$.

The theoretical comparison between the existing and proposed estimators, has been performed in this section, and the conditions of better performance of $\bar{Y}_{FTc}$ have been derived.

**[A]:** The variance of $\bar{Y}_{ac}$ in SRSWOR is given by

$$V(\bar{Y}_{ac}) = \frac{N-n}{Nn} S_{wy}^2$$

and the MSE of $\bar{Y}_{FTc}$ is

$$M(\bar{Y}_{FTc})_{min} = \frac{S_{wy}^2}{n}(1-\rho_{wyx}^2)$$

Now, let

$$D_1 = V(\bar{Y}_{ac}) - M(\bar{Y}_{FTc})_{min}$$
$$= \frac{N-n}{Nn} S_{wy}^2 - \frac{S_{wy}^2}{n}(1-\rho_{wyx}^2)$$

$\bar{Y}_{FTc}$ is better than $\bar{Y}_{ac}$ if $D_1 > 0$

i.e.
$$\frac{N-n}{Nn} S_{wy}^2 - \frac{S_{wy}^2}{n}(1-\rho_{wyx}^2) > 0$$
$$\frac{N-n}{N} > (1 - \rho_{wyx}^2)$$
$$n < N\rho_{wyx}^2$$

If the condition $n < N\rho_{wyx}^2$ holds, then $\bar{Y}_{FTc}$ is always better than $\bar{Y}_{ac}$.

**[B]:** The mean squared error of $\bar{Y}_{R_{ac}}$ is

$$\text{MSE}(\bar{Y}_{R_{ac}}) = \frac{\bar{Y}^2}{n}\left[C_{wy}^2 + C_{wx}^2 - 2\rho_{wyx}C_{wy}C_{wx}\right]$$

Let $D_2 = \left[M\left(\bar{Y}_{R_{ac}}\right) - M(\bar{Y}_{FTc})_{min}\right]$

$$= \frac{\bar{Y}^2}{n}\left[C_{wy}^2 + C_{wx}^2 - 2\rho_{wyx}C_{wy}C_{wx}\right] - \frac{\bar{Y}^2}{n}\left[C_{wy}^2 - C_{wy}^2\rho_{wyx}^2\right]$$

$\bar{Y}_{FTc}$ is better than $\bar{Y}_{R_{ac}}$, if $D_2 > 0$, i.e. $\rho_{wyx} < \frac{C_{wx}}{C_{wy}}$.

If the above condition satisfies then $\bar{Y}_{FTc}$ is better than $\bar{Y}_{Rac}$.

**[C]:** The mean squared error of $\bar{Y}_{R_{ac1}}$ is

$$M(\bar{Y}_{R_{ac1}}) = \frac{\bar{Y}^2}{n}\left[C_{wy}^2 + \alpha^2 C_{wx}^2 - 2\alpha\rho_{wyx}C_{wy}C_{wx}\right]$$

Let $D_3 = \left[M\left(\bar{Y}_{R_{ac1}}\right) - M(\bar{Y}_{FTc})_{min}\right]$

$$= \frac{\bar{Y}^2}{n}\left[C_{wy}^2 + \alpha^2 C_{wx}^2 - 2\alpha\rho_{wyx}C_{wy}C_{wx}\right] - \frac{\bar{Y}^2}{n}\left[C_{wy}^2 - C_{wy}^2\rho_{wyx}^2\right]$$

$\bar{Y}_{FTc}$ is better than $\bar{Y}_{R_{ac1}}$, if $D_3 > 0$

i.e. $\rho_{wyx} < \alpha\frac{C_{wx}}{C_{wy}}$

If the above condition satisfies then $\bar{Y}_{FTc}$ is better than $\bar{Y}_{R_{ac1}}$.

**[D]:** The mean squared error of $\bar{Y}_{R_{ac2}}$ is

$$M(\bar{Y}_{R_{ac2}}) = \frac{\bar{Y}^2}{n}\left[C_{wy}^2 + \omega^2 C_{wx}^2 - 2\omega\rho_{wyx}c_{wy}c_{wx}\right]$$

Let $D_4 = \left[M\left(\bar{Y}_{R_{ac2}}\right) - M(\bar{Y}_{FTc})_{min}\right]$

$$= \frac{\bar{Y}^2}{n}\left[C_{wy}^2 + \omega^2 C_{wx}^2 - 2\omega\rho_{wyx}c_{wy}c_{wx}\right] - \frac{\bar{Y}^2}{n}\left[C_{wy}^2 - C_{wy}^2\rho_{wyx}^2\right]$$

$\bar{Y}_{FTc}$ is better than $\bar{Y}_{\mathrm{R}_{\mathrm{ac2}}}$, if $D_4 > 0$

i.e. $\rho_{wyx} < \omega\frac{C_{wx}}{C_{wy}}$

If above condition satisfies then $\bar{Y}_{FTc}$ is better than $\bar{Y}_{R_{ac2}}$.

**[E]:** The mean squared error of $\bar{Y}_{R_{ac3}}$ is

$$M(\bar{Y}_{R_{ac3}}) = \frac{\bar{Y}^2}{n}\left[C_{wy}^2 + \delta^2 C_{wx}^2 - 2\delta\rho_{wyx}C_{wy}C_{wx}\right]$$

Let $D_5 = \left[M\left(\bar{Y}_{R_{ac2}}\right) - M(\bar{Y}_{FTc})_{min}\right]$

$$= \frac{\bar{Y}^2}{n}\left[C_{wy}^2 + \delta^2 C_{wx}^2 - 2\delta\rho_{wyx}C_{wy}C_{wx}\right] - \frac{\bar{Y}^2}{n}\left[C_{wy}^2 - C_{wy}^2\rho_{wyx}^2\right]$$

$\bar{Y}_{FTc}$ is better than $\bar{Y}_{R_{ac3}}$, if $D_5 > 0$

i.e. $\rho_{wyx} < \delta\frac{C_{wx}}{C_{wy}}$

If the above condition satisfies then $\bar{Y}_{FTc}$ is better than $\bar{Y}_{R_{ac3}}$.

## 6. Empirical study

Appendix A displays an engendered simulated population covering amounts of $y$ and $x$ respectively. Summary of population is calculated as

$N = 400$ $\quad \bar{Y} = 1.2275$ $\quad \bar{X} = 0.56500$ $\quad S_{wy}^2 = 12.6791$ $\quad S_{wx}^2 = 3.79790$
$C_{wy} = 2.9008$ $\quad C_{wx} = 3.44920$ $\quad \rho_{wxy} = 0.80710$ $\quad \beta_2(w_x) = 92.63470$
$\beta_2(w_y) = 23.0357$ $\quad S_{wxy} = 5.60070$ $\quad V = 0.67877$

Taking random sample of size 5, 10, 20 and 40 by SRSWOR and, by solving optimum condition (4.6) i.e., the equation of degree 3 in terms of $k$ we got three $k$-values for different sample sizes as shown below in Table 6.1.

**Table 6.1:** Values of $k$ for different sample sizes

| Sample Size | $k_1$ | $k_2$ | $k_3$ |
|---|---|---|---|
| $n = 5$ | 7.1827013 | 1.8390406 | 2.1566182 |
| $n = 10$ | 7.2310521 | 1.7829198 | 2.2299408 |
| $n = 20$ | 7.3283849 | 1.7091097 | 2.3370733 |
| $n = 40$ | 7.5273670 | 1.6156457 | 2.4928638 |

Table 6.1 reveals that for sample size $n = 5$, by simplifying the optimality condition (4.6) we obtained a cubic equation of variable $k$ and using the available constants the three values of $k$ are $k_1$, $k_2$ and $k_3$. A similar procedure is used for sample sizes 10, 20 and 40.

## 7. Simulation study

In this section, we conducted a simulation using the population data of appendix A. The population was visualized through appendix A, and data were provided to describe the population. The simulation process involved the following method.

A preliminary sample of $n$ units was carefully chosen by SRSWOR. Once the preliminary sample was selected, the y-values and x-values were obtained for each unit in the sample. The criterion for inclusion of items in the sample is $L = \{y: y > 0\}$. Every estimator was obtained for 5,000 iterations. The estimators were then applied to these samples to estimate population parameters of interest. By repeating this process, the study aimed to obtain accuracy estimates for the estimators.

The determination of this simulation is likely to assess the presentation of different estimators under adaptive sampling (ACS) scheme using various preliminary sample sizes. The bias is calculated by the formula

$$B(\hat{y}) = \frac{1}{5000}\sum_{i=1}^{5000}[(\hat{y}_{\mathrm{i}}) - \bar{Y}] \tag{7.1}$$

and the mean squared error is calculated by the formula

$$M(\hat{y}) = \frac{1}{5000}\sum_{i=1}^{5000}[(\hat{y}_{\mathrm{i}}) - \bar{Y}]^2 \tag{7.2}$$

The size of the preliminary sample is considered as $n = 5$, 10, 20 and 40 and repeated 5,000 times for each and every sample size $n$.

**Table 7.1:** Bias and MSE of Existing and Proposed Estimators

| $\hat{y}$ | **Bias** ($\hat{y}$) | | | | **MSE** ($\hat{y}$) | | | |
|---|---|---|---|---|---|---|---|---|
| | n = 5 | n = 10 | n = 20 | n = 40 | n = 5 | n = 10 | n = 20 | n = 40 |
| $\bar{Y}_{ac}$ | 0 | 0 | 0 | 0 | 2.53582 | 1.26791 | 0.63395 | 0.31697 |
| $\bar{Y}_{R_{ac}}$ | -0.25727 | -0.24622 | -0.25663 | -0.27817 | 0.35221 | 0.26310 | 0.12222 | 0.11167 |
| $\bar{Y}_{R_{ac1}}$ | 1.61925 | 1.78461 | 1.89083 | 1.88892 | 4.87287 | 4.41820 | 4.18421 | 4.04436 |
| $\bar{Y}_{R_{ac2}}$ | 3.94233 | 3.85695 | 3.87888 | 3.88283 | 30.22787 | 21.38539 | 18.33143 | 17.80784 |
| $\bar{Y}_{R_{ac3}}$ | -0.21504 | -0.19859 | -0.20757 | -0.28289 | 0.34885 | 0.24523 | 0.10370 | 0.11321 |
| $\bar{Y}_{FTc_1}$ | 0.03349 | 0.08942 | 0.11107 | 0.12835 | 0.32494 | 0.23383 | 0.10168 | 0.07645 |
| $\bar{Y}_{FTc_2}$ | 0.21346 | 0.30079 | 0.36296 | 0.41172 | 0.48638 | 0.39157 | 0.25440 | 0.27200 |
| $\bar{Y}_{FTc_3}$ | -0.34290 | -0.54661 | -0.78513 | -1.56488 | 0.57626 | 0.63471 | 0.91916 | 3.16843 |

By observing Table 7.1 the proposed estimator $\bar{Y}_{FTc}$ has minimum mean squared error for $k = k_1$ and minimum bias as well. The proposed estimator is better over all the estimators under consideration and the estimator $\bar{Y}_{FTc_1}$ is best overall.

## 8. Discussion and conclusion

In the present manuscript some estimators are discussed with their properties using the concept of large sample approximations in adaptive cluster sampling [see Chutiman (2013)] and discussed about factor-type estimator of Singh and Shukla (1987), Shukla and Thakur (2008), etc. Then raising idea from these all we experimented on the factor-type estimator under the same sampling design and found that the factor-type estimator of Singh and Shukla (1987) performed excellently overall in the adaptive cluster sampling design. The bias and mean squared error (*MSE*) of factor-type estimator are obtained up to the first order in terms of population parameters. The condition of optimality is derived as well.

From the results of simulation (table 7.1), it is clear that modified ratio estimators in ACS design have more bias and mean squared error as compared to the factor-type estimator at optimum value of $k$, i.e. $\bar{Y}_{FTc_1}$, $\bar{Y}_{FTc_2}$ and $\bar{Y}_{FTc_3}$. Also, it is proved that the proposed estimator is closer to the true value of average cases. The proposed estimator $\bar{Y}_{FTc_1}$ results in the lowest MSE as compared to all the estimators considered in this article. This proves that the proposed factor-type estimator has a greater efficiency than all the estimators under consideration.

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## Appendix A: Population for Empirical Study

Observations of Study Variable (Y)

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 2 | 24 | 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 22 | 5 | 4 | 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 2 | 0 | 4 | 8 | 0 | 0 | 0 | 0 | 0 | 33 | 0 | 0 | 0 | 27 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7 | 6 | 7 | 1 | 0 | 5 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 21 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4 | 0 | 5 | 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 5 | 7 | 0 | 7 | 7 | 6 | 3 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 5 | 4 | 3 | 0 | 5 | 8 | 4 | 5 | 1 | 0 | 5 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 7 | 65 | 0 | 4 | 5 | 0 | 9 | 0 | 0 | 0 | 0 | 0 | 3 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 4 | 5 | 0 | 7 | 3 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 27 | 0 | 0 | 21 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 29 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Corresponding Observations of Auxiliary Variable (X)

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 11 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 11 | 2 | 2 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 0 | 2 | 4 | 0 | 0 | 0 | 0 | 0 | 12 | 0 | 0 | 0 | 15 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3 | 2 | 3 | 0 | 0 | 2 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 16 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 0 | 2 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 0 | 3 | 3 | 2 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 2 | 2 | 1 | 0 | 2 | 3 | 2 | 2 | 0 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 3 | 18 | 0 | 2 | 2 | 0 | 4 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 2 | 2 | 0 | 3 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 12 | 0 | 0 | 12 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 27 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |